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A hybrid Green's function method for the hyperbolic heat conduction problems

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ABSTRACT

The present study is devoted to propose a hybrid Green's function method to investigate the hyperbolic heat conduction problems. The difficulty of the numerical solutions of hyperbolic heat conduction problems is the numerical oscillation in the vicinity of sharp discontinuities. In the present study, we have developed a hybrid method combined the Laplace transform, Green's function and ε-algorithm acceleration method for solving time dependent hyperbolic heat conduction equation. From one- to three-dimensional problems, six different examples have been analyzed by the present method. It is found from these examples that the present method is in agreement with the Tsai-tse Kao's solutions [Tsai-tse Kao, Non-Fourier heat conduction in thin surface layers, J. Heat Transfer 99 (1977) 343–345] and does not exhibit numerical oscillations at the wave front. The propagation of the two- and three-dimensional thermal wave becomes so complicated because it occur jump discontinuities, reflections and interactions in these numerical results of the problem and it is difficult to find the analytical solutions or the result of other study to compare with the solutions of the present method.

1. Introduction

In recent years, study of the hyperbolic heat conduction has received considerable interest, because of it wide applicability in engineering applications, such as laser-aided material processing, cryogenic engineering, the high-intensity electromagnetic irradiation of a solid and the high-rate heat transfer in rarefied media. The solutions of the hyperbolic heat conduction can be found in a number of publications such as Tsai-tse Kaov [1] studied the HHC in thin surface layers obtained an analytical solution. Baumeister and Hamill [2], Taitel [3], Ozisik and Vick [4], and Wu [5] obtained an analytical solution of one-dimensional HHC, for a semi-infinite medium or in a finite medium with convection, or radiation at the wall surface. Carey and Tai [6] applied the central and backward difference schemes to examine the oscillation of numerical solution at the reflected boundary. To remedy the numerical difficulty encountered, many numerical schemes have been proposed such as the predictor–corrector scheme [7], the transfinite element formulation [8], and a technique based on the Galerkin finite element and mixed implicit–explicit scheme [9], the characteristic method [10], and the hybrid scheme [11]. The effect of the surface radiation on thermal wave propagation in a one-dimensional slab has been studied by Glass et al. [12] and Yeung and Tung [13] and two-dimensional solutions are given by Yang [14], Chen and Lin [15] and Shen [16]. The problem of the HHC in thin surface layers has been investigated by Chen [17]. Loh et al. [18] investigated the problems of the fast transient Fourier and Non-Fourier heat conduction problems.

The purpose of the present study is to propose a hybrid method investigating the hyperbolic heat conduction problem. The present method combine the Laplace transform, Green's function and the ε-algorithm acceleration method for solving time dependent hyperbolic heat conduction equation. The Laplace transfer method is used to remove the time-dependent terms from the governing equation, and then the s-domain dimensionless temperature function is obtained by the Green's function scheme. Finally, the time-domain dimensionless temperature can be determined by the numerical inversion of the Laplace transform and the ε-algorithm acceleration method. It is found that the present method is in agreement with the analytical solutions [1] and does not exhibit numerical oscillations at the wave front.

2. Analysis

Consider the problems of hyperbolic heat conduction. The three-dimensional hyperbolic heat conduction equation is given by

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2} \tag{1}$$

And boundary condition $k_i\frac{\partial T}{\partial n_i}+h_iT=f_i(r,t)$ on S_i

For convenience of numerical analysis, let us define by the following dimensionless variables

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Nomenclature

Bi	Biot number, $\frac{hl}{k}$
C	propagation velocity of thermal wave
c_p	specific heat
f_r	reference heat flux
G	Green's function
h	thermal convection coefficient
k	thermal conductivity
n	outward-drawn normal vector to the boundary surface
$\widehat{n}$	dimensionless outward-drawn normal vector to the boundary surface
q	heat flux
Q	dimensionless heat flux, $\frac{q}{f_r}$
S_i	boundary surface
s	Laplace transform parameter
sum	summation of a series of function
T	temperature
T_0	surrounding temperature
x, y, z	coordinators
Greek letters	
α	thermal diffusivity, $\frac{k}{\rho c_p}$
η	dimensionless length, $\frac{Cx}{2\alpha}$
θ	dimensionless temperature, $\frac{(T-T_0)kc}{\alpha f_r}$
ρ	density
ε	ε-algorithm parameter
ς	dimensionless length, $\frac{Cz}{2\alpha}$
ξ	dimensionless time, $\frac{C^2t}{2\alpha}$
ζ	dimensionless length, $\frac{Cy}{2\alpha}$
Superscript	
–	The Laplace transform

$$\xi = \frac{C^2 t}{2\alpha} \tag{2}$$

$$\eta = \frac{Cx}{2\alpha} \quad \zeta = \frac{Cy}{2\alpha} \quad \varsigma = \frac{Cz}{2\alpha} \tag{3}$$

$$\theta(\eta, \xi) = \frac{kC(T - T_0)}{\alpha f_r} \tag{4}$$

$$Q(\eta, \xi) = \frac{q}{f_r} \tag{5}$$

The resulting equation becomes

$$\frac{\partial^2 \theta}{\partial \xi^2} + 2\frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \zeta^2} + \frac{\partial^2 \theta}{\partial \varsigma^2} \tag{6}$$

And boundary condition $\frac{\partial \theta}{\partial \widehat{n}} + Bi\theta = \widehat{f}_i$ on S_i

3. Numerical scheme

In all illustrative examples of this study, the dimensionless initial conditions are given as

$$\theta(\eta, \zeta, \varsigma, 0) = 0, \text{ and } \frac{\partial \theta}{\partial \xi}(\eta, \zeta, \varsigma, 0) = 0 \tag{7}$$

To remove the ξ-dependent terms, taking the Laplace transform of Eq. (6) with respect to ξ gives

$$\frac{\partial^2 \bar{\theta}}{\partial \eta^2} + \frac{\partial^2 \bar{\theta}}{\partial \zeta^2} + \frac{\partial^2 \bar{\theta}}{\partial \varsigma^2} - (s^2 + 2s)\bar{\theta} = 0 \tag{8}$$

And boundary condition $\frac{\partial \bar{\theta}}{\partial \widehat{n}} + Bi\bar{\theta} = \bar{\widehat{f}}_i$ on S_i

To solve the above s-domain heat conduction problem we consider the following auxiliary problem for the same region

$$\frac{\partial^2 \overline{G}}{\partial \eta^2} + \frac{\partial^2 \overline{G}}{\partial \zeta^2} + \frac{\partial^2 \overline{G}}{\partial \varsigma^2} + \delta(r - r_0) - (s^2 + 2s)\overline{G} = 0 \tag{9}$$

$$\frac{\partial \overline{G}}{\partial \widehat{n}} + Bi\overline{G} = 0, \text{ on } S_i$$

The Green's function $\overline{G}(r, s|r_0, s)$ is determined from Eq. (9) by the method of separating variables [19], and we obtain the s-domain solution $\bar{\theta}(r, s)$ of the heat conduction problem, Eq. (8) in terms of the Green's function $\overline{G}(r, s|r_0, s)$ as

$$\bar{\theta}(r, s) = \sum_{i=1}^{S} \int_{S_i} \overline{G}(r, s|r_0, s)|_{r_0 = r_i} \bar{\widehat{f}}(r, s) dS_i \tag{10}$$

where S_i refers to the boundary surface S_i of the region R, i = 1, 2, 3, ... S and S in number continuous boundary surfaces.

The dimensionless temperature $\theta(r, \xi)$ can be determined by the numerical inversion of the Laplace transform and the ε-algorithm acceleration method.

For a non-monotonous $f_N(t) = \sum_{k=1}^{N} u_k$, the ε-algorithm acceleration convergence method is expressed as

Let $N = 2q + 1$, $q \in N$,

$$sum_m = \sum_{k=1}^{N} u_k \tag{11}$$

$$\text{And } \varepsilon_{p+1}^{(m)} = \varepsilon_{p-1}^{(m+1)} + \frac{1}{\left(\varepsilon_p^{(m+1)} - \varepsilon_p^{(m)}\right)}, \varepsilon_0^{(m)} = 0, \quad \varepsilon_1^{(m)} = sum_m \tag{12}$$

Then the sequence $\varepsilon_1^{(1)}, \varepsilon_3^{(1)}, \varepsilon_5^{(1)}, \ldots, \varepsilon_{2q+1}^{(1)} = \varepsilon_N^{(1)}$, converges to $f_\infty(t)$

4. Results and discussion

Example 1. One-dimensional problem prescribed wall temperature. The initial and boundary conditions for this case are given by

$$\theta(\eta, 0) = 0, \quad \frac{\partial \theta}{\partial \xi}(\eta, 0) = 0 \tag{13}$$

$$\theta(0, \xi) = 1, \quad \theta(\eta \to \infty, \xi) = 0 \tag{14}$$

The $\theta(\eta, s)$ is obtained as

$$\bar{\theta}(\eta, s) = \sum_{n=1}^{\infty} \frac{2n\pi}{s[(n\pi)^2 + s^2 + 2s]} \sin(n\pi\eta) \tag{15}$$

The Tsai-tse Kao's solution [1] of this example is expressed as

$$\theta(\eta, \xi) = e^{\frac{-\eta}{2\varepsilon}} \left\{ e^{-\eta} + \left(1 - \frac{\varepsilon^2}{4}\right)^{\frac{1}{2}} \eta \int_\eta^\xi e^{-\tau} \frac{I_1\left\{\left[\left(1 - \frac{\varepsilon^2}{4}\right)(\tau^2 - \eta^2)\right]^{\frac{1}{2}}\right\}}{(\tau^2 - \eta^2)^{\frac{1}{2}}} d\tau \right\} U(\xi - \eta) \tag{16}$$

Table 1 lists the comparison of the present method solutions and analytical solutions for the problem at ξ = 0.5 and ξ = 0.8. From Table 1, it is seen that the present method solutions are in agreement with the analytical solution using the Eq. (16).

Example 2. One-dimensional problem prescribed wall heat flux. The initial and boundary conditions for this case are given by

$$\theta(\eta, 0) = 0, \quad \frac{\partial \theta(\eta, 0)}{\partial \xi} = 0 \tag{17}$$

$$Q(0, \xi) = 1, \quad Q(\eta \to \infty, \xi) = 0 \quad Q(\eta, 0) = 0 \tag{18}$$

Table 1
Comparison of the present method and analytical solution resulting from a prescribed wall temperature.

Present method			Analytic solution Eq. (16) $\varepsilon = 0$	
x	$\xi = 0.5$	$\xi = 0.8$	$\xi = 0.5$	$\xi = 0.8$
0.0	1.000000	1.000000	1.000000	1.000000
0.1	0.919904	0.928205	0.919913	0.928184
0.2	0.840201	0.856537	0.840177	0.856657
0.3	0.761172	0.785696	0.761140	0.785709
0.4	0.683119	0.715850	0.683146	0.715623
0.5	0.303307	0.646677	0.303265	0.646676
0.6	0.000029	0.579426	0.000000	0.579140
0.7	0.000008	0.513275	0.000000	0.513275
0.8	0.000026	0.224633	0.000000	0.224664
0.9	0.000060	0.000564	0.000000	0.000000
1.0	0.000000	0.000000	0.000000	0.000000

The boundary condition for the Laplace transform of the dimensionless temperature at surface $\eta = 0$ can be obtained

$$\frac{d\bar{\theta}}{d\eta}(0,s) = -\frac{s+2}{s} \tag{19}$$

The $\theta(\eta, s)$ is obtained as

$$\bar{\theta}(\eta,s) = \sum_{n=1}^{\infty} \frac{2(s+2)}{s\left[\left(\frac{(2n-1)\pi}{2}\right)^2 + s^2 + 2s\right]} \cos\left(\frac{(2n-1)\pi}{2}\eta\right) \tag{20}$$

Fig. 1 represents the influence of the one-dimensional problem on hyperbolic heat conduction with a prescribed wall heat flux at $\xi = 0.125$, $\xi = 0.25$, $\xi = 0.5$, and $\xi = 0.75$. It can be seen that the present method solutions do not exhibit numerical oscillations at the wave front.

Example 3. One-dimensional problem prescribed in a finite slab. The initial and boundary conditions for this case are given by

$$\theta(\eta,0) = 0, \quad \frac{\partial\theta}{\partial\xi}(\eta,0) = 0 \tag{21}$$

$$\theta(0,\xi) = 1, \quad \frac{\partial\theta}{\partial\eta}(1,\xi) = 0 \tag{22}$$

The $\theta(\eta, s)$ is obtained as

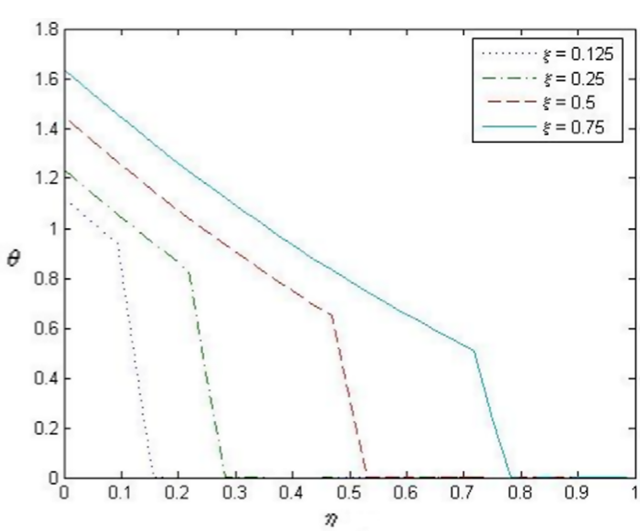

Fig. 1. The influence of the one-dimensional problem on hyperbolic heat conduction with a prescribed wall heat flux at $\xi = 0.125$, $\xi = 0.25$, $\xi = 0.5$, and $\xi = 0.75$.

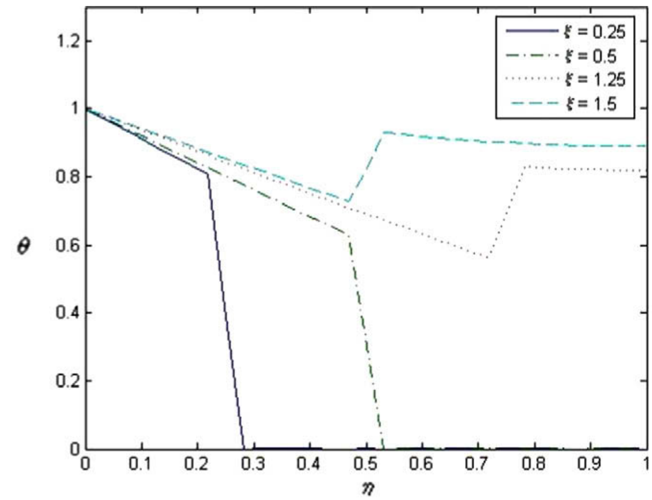

Fig. 2. The effect of the one-dimensional problem on hyperbolic heat conduction at $\xi = 0.25$, $\xi = 0.5$, $\xi = 1.25$, and $\xi = 1.5$.

$$\bar{\theta}(\eta,s) = \sum_{n=1}^{\infty} \frac{2(n-1)\pi}{s\left[\left(\frac{(2n-1)\pi}{2}\right)^2 + s^2 + 2s\right]} \sin\left(\frac{(2n-1)\pi}{2}\eta\right) \tag{23}$$

Fig. 2 illustrates the effect of the one-dimensional problem on hyperbolic heat conduction at $\xi = 0.25$, $\xi = 0.5$, $\xi = 1.25$, and $\xi = 1.5$ that the thermal wave occur jump discontinuities, reflections and interactions in this problem.

Example 4. Two-dimensional problem prescribed wall temperature. The initial and boundary conditions for this case are given by

$$\theta(\eta,\zeta,0) = 0, \quad \frac{\partial\theta(\eta,\zeta,0)}{\partial\xi} = 0 \tag{24}$$

$$\theta(0,\zeta,\xi) = 1, \quad \theta(1,\zeta,\xi) = 0 \tag{25}$$

$$\theta(\eta,0,\xi) = 1, \quad \theta(\eta,1,\xi) = 0 \tag{26}$$

The $\theta(\eta, \zeta, s)$ is obtained as

$$\bar{\theta}(\eta,\zeta,s) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4[1-(-1)^m]n}{ms[(n\pi)^2 + (m\pi)^2 + s^2 + 2s]} \sin(n\pi\eta) \times \sin(m\pi\zeta) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4[1-(-1)^n]m}{ns[(n\pi)^2 + (m\pi)^2 + s^2 + 2s]} \times \sin(n\pi\eta)\sin(m\pi\zeta) \tag{27}$$

Fig. 3 shows the three-dimensional sketch of dimensionless temperature for (a) $\xi = 0.25$, (b) $\xi = 0.5$, and (c) $\xi = 0.75$ that the η- and ζ-direction thermal wave interact before the jump discontinuities.

Example 5. Two-dimensional problem prescribed in a plate. The initial and boundary conditions for this case are given by

$$\theta(\eta,\zeta,0) = 0, \quad \frac{\partial\theta(\eta,\zeta,0)}{\partial\xi} = 0 \tag{28}$$

$$\theta(0,\zeta,\xi) = 1, \quad \frac{\partial\theta(1,\zeta,\xi)}{\partial\eta} = 0 \tag{29}$$

$$\theta(\eta,0,\xi) = 1, \quad \frac{\partial\theta(\eta,1,\xi)}{\partial\zeta} = 0 \tag{30}$$

The $\theta(\eta, \zeta, s)$ is obtained as

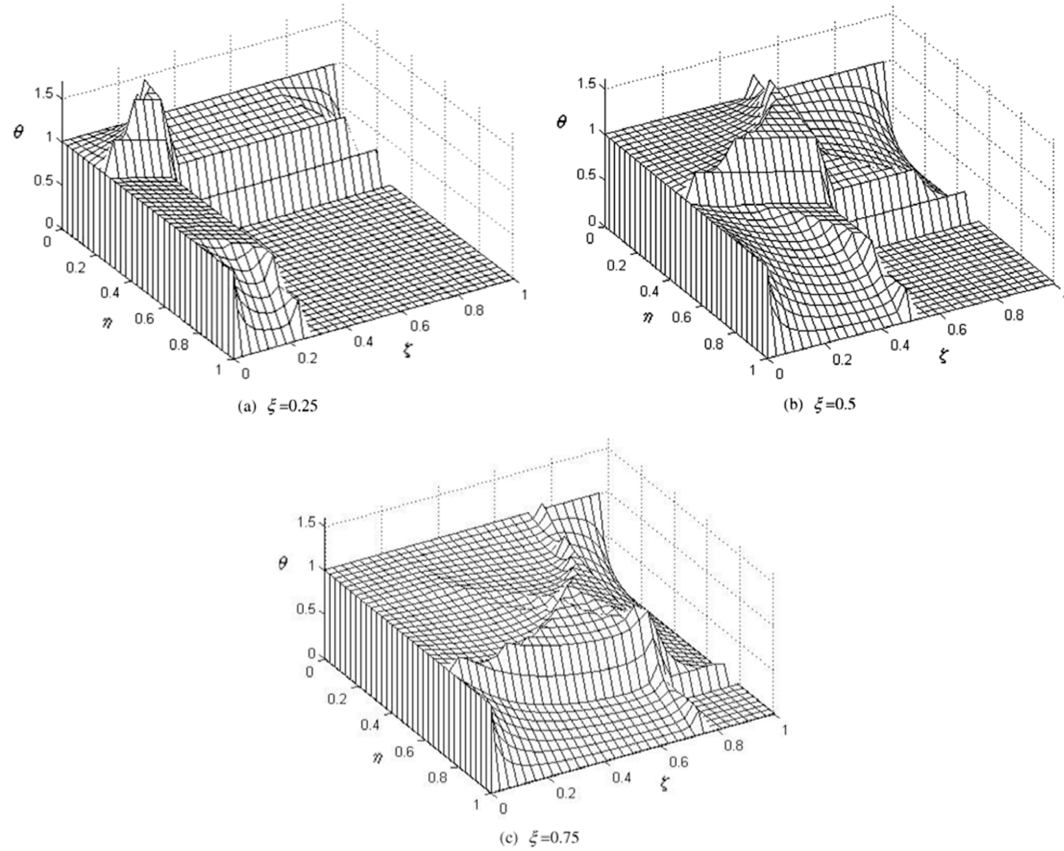

Fig. 3. The three-dimensional sketch of dimensionless temperature for (a) $\xi = 0.25$, (b) $\xi = 0.5$, and (c) $\xi = 0.75$.

$$\begin{aligned}\bar{\theta}(\eta,\zeta,s) = {} & \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4(2n-1)}{(2m-1)\left[\left(\frac{2n-1}{2}\pi\right)^2+\left(\frac{2m-1}{2}\pi\right)^2+s^3+2s^2\right]} \\ & \times \sin\left(\frac{(2n-1)}{2}\pi\eta\right)\sin\left(\frac{(2m-1)}{2}\pi\zeta\right) \\ & + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4(2m-1)}{(2n-1)\left[\left(\frac{2n-1}{2}\pi\right)^2+\left(\frac{2m-1}{2}\pi\right)^2+s^3+2s^2\right]} \\ & \times \sin\left(\frac{(2n-1)}{2}\pi\eta\right)\sin\left(\frac{(2m-1)}{2}\pi\zeta\right) \end{aligned} \quad (31)$$

Fig. 4 shows the three-dimensional sketch of dimensionless temperature for (a) $\xi = 0.5$, (b) $\xi = 0.75$, (c) $\xi = 1.2$, and (d) $\xi = 1.5$ that the η- and ζ-direction thermal wave interact before the jump discontinuities and after reflections.

Example 6. Three-dimensional problem prescribed in cubic solid. The initial and boundary conditions for this case are given by

$$\theta(\eta,\zeta,\varsigma,0) = 0, \quad \frac{\partial\theta(\eta,\zeta,\varsigma,0)}{\partial\xi} = 0 \quad (32)$$

$$\theta(0,\zeta,\varsigma,\xi) = 1, \quad \frac{\partial\theta(1,\zeta,\varsigma,\xi)}{\partial\eta} = 0 \quad (33)$$

$$\theta(\eta,0,\varsigma,\xi) = 1, \quad \frac{\partial\theta(\eta,1,\varsigma,\xi)}{\partial\zeta} = 0 \quad (34)$$

$$\theta(\eta,\zeta,0,\xi) = 1, \quad \frac{\partial\theta(\eta,\zeta,1,\xi)}{\partial\varsigma} = 0 \quad (35)$$

The $\theta(\eta,\zeta,\varsigma,s)$ is obtained as

$$\begin{aligned}&\bar{\theta}(\eta,\zeta,\varsigma,s) \\ &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{16(2n-1)}{(2m-1)(2p-1)\left[\left(\frac{2n-1}{2}\pi\right)^2+\left(\frac{2m-1}{2}\pi\right)^2+\left(\frac{2p-1}{2}\pi\right)^2+s^3+2s^2\right]\pi} \\ &\times \sin\left(\frac{(2n-1)}{2}\pi\eta\right)\sin\left(\frac{(2m-1)}{2}\pi\zeta\right)\sin\left(\frac{(2p-1)}{2}\pi\varsigma\right) \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{16(2m-1)}{(2n-1)(2p-1)\left[\left(\frac{2n-1}{2}\pi\right)^2+\left(\frac{2m-1}{2}\pi\right)^2+\left(\frac{2p-1}{2}\pi\right)^2+s^3+2s^2\right]\pi} \\ &\times \sin\left(\frac{(2n-1)}{2}\pi\eta\right)\sin\left(\frac{(2m-1)}{2}\pi\zeta\right)\sin\left(\frac{(2p-1)}{2}\pi\varsigma\right) \\ &+ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{16(2p-1)}{(2m-1)(2n-1)\left[\left(\frac{2n-1}{2}\pi\right)^2+\left(\frac{2m-1}{2}\pi\right)^2+\left(\frac{2p-1}{2}\pi\right)^2+s^3+2s^2\right]\pi} \\ &\times \sin\left(\frac{(2n-1)}{2}\pi\eta\right)\sin\left(\frac{(2m-1)}{2}\pi\zeta\right)\sin\left(\frac{(2p-1)}{2}\pi\varsigma\right) \end{aligned} \quad (36)$$

Fig. 5 shows the three-dimensional sketch of dimensionless temperature for $\xi = 0.5$ (a) $\varsigma = 0.75$, and (b) $\varsigma = 0.25$ that the η-, ζ- and ς-direction thermal waves interact before the jump discontinuities. For (a) $\varsigma = 0.75$ is after ς-direction thermal waves jump discontinuity ($\varsigma = 0.5$), so it is same as the 2-D problem example 5 Fig. 4(a).

5. Conclusions

The hybrid method has shown success in solving the hyperbolic heat conduction problem. To illustrate the accuracy and efficiency of the method, from one- to three-dimensional problems, six different examples have been analyzed. It is found from these examples that the present method is in agreement with the analytical solutions [1] and does not exhibit numerical oscillations at the

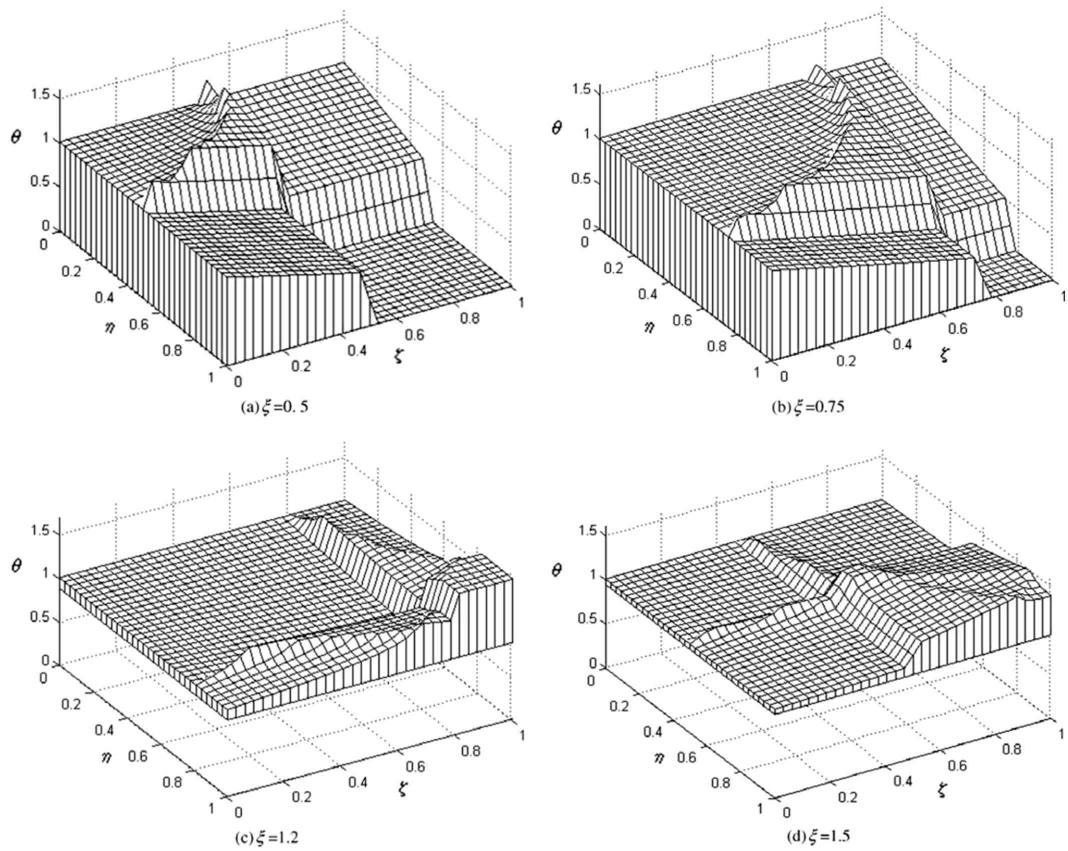

Fig. 4. The three-dimensional sketch of dimensionless temperature for (a) ξ = 0. 5, (b) ξ = 0.75, (c) ξ = 1.2, and (d) ξ = 1.5.

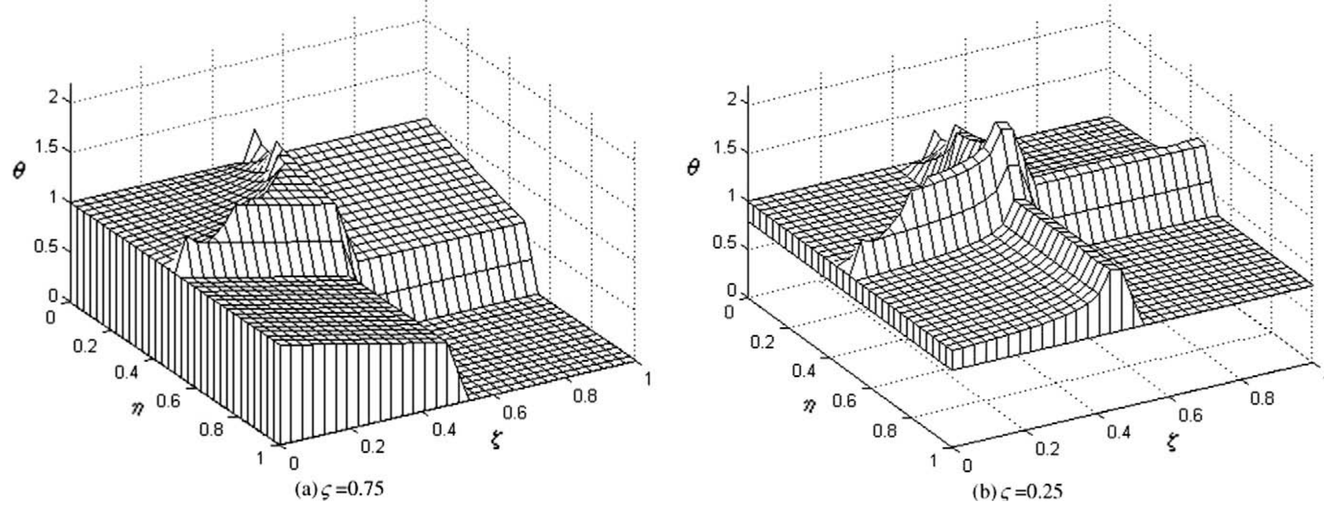

Fig. 5. The three-dimensional sketch of dimensionless temperature for ξ = 0.5 (a) ς = 0.75 and (b) ς = 0.25.

wave front and the propagation of the two- and three-dimensional thermal wave becomes so complicated because it occur jump discontinuities, reflections and interactions in these numerical results of the hyperbolic heat conduction problems.

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